

Dynamics of an oscillating Stirling heat pump



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HIGHLIGHTS

- The working mode of an oscillating Stirling heat pump is described.
- A methodology has been developed for preliminary designs.
- An optimal working condition has been identified.
- Stability of operation has been explored considering changes in working conditions.
- A startup logic that avoids the use of complementary volumes or additional devices has been proposed.

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ABSTRACT

In this paper, a promising heat pump based on Stirling technology is examined. Based on the volume variation produced by the oscillating displacement of two pistons, the advantages of this particular system lie in the elimination of a transmission mechanism, as the driving machines are directly coupled to the working pistons, and in the use of environmentally friendly working gases. The system is analysed by coupling the dynamics to an isothermal model of the Stirling cycle. A methodology is proposed to preliminarily size this type of heat pump based on simulations performed on a mathematical model built in MATLAB. The stability of operation is analysed by considering possible changes in working conditions. Actions are proposed to minimize the effect of these changes on the performance of the system.

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1. Introduction

More commonly known as a power producing heat engine, the Stirling cycle can also operate in its reverse mode as a heat pump. Conventional heat pump systems offer economical alternatives that provide energy efficiency and environmental benefits by reducing energy consumption and associated greenhouse gas emissions [1]. Although a significant amount of work has been performed on the prime mover applications of the Stirling cycle, the use of the cycle as a heat pump has not been explored in depth. Stirling Thermal Motors, STM, developed a conceptual design of a heat pump application in 2000 based on the 25 kW STM4-120 Stirling engine [2]. A prototype was not built, but the expected heating capacity of the system was 15 kW with better performance than the solutions on the market. The most remarkable work in this field was the prototype developed by Haywood [3] in 2004. A heat

pump prototype was developed and tested, obtaining performance similar to products on the market.

To achieve high efficiency Stirling solutions and avoid the high cost and time-consuming trial and error process, designers usually model the system during the design phase. The first analytical model for Stirling applications was introduced by Schmidt in 1871. Since then, different authors have worked on this area to obtain accurate models, based on the thermodynamic cycle and analysis of the influence of the different components and working parameters. Some of these works focus on the transmission mechanism and the machine dynamics. Cheng [4] introduced the concept of mechanism effectiveness into the theoretical model and studied the optimal combination of the phase angle and swept volume ratio. Dynamics of Stirling solutions have been studied mainly in free piston applications to understand the behaviour of these applications and the influence of geometrical parameters on them. Usually, the dynamics and thermodynamics are coupled, and the equations describing the system are linearized [5,6].

The advantages of the proposed system lie in the elimination of a transmission mechanism, as the driving machines are directly

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Nomenclature

A	area of the piston palette (m^2)	V_{swe}	expansion space swept volume (m^3)
d	distance to the rotation axis of the piston (m)	V_{D}	volume that is unswept by the pistons (m^3)
f	frequency (Hz)	R	gas constant (J/kg K)
I	inertia (kg m^2)	R_1	piston inner radio (m)
L	piston length (m)	R_2	piston outer radio (m)
m_{t}	working gas total mass (kg)	V_{D}	dead volume (m^3)
T_{h}	hot space temperature (K)	ω	angular frequency (rad/s)
T_{k}	cold space temperature (K)	W	mechanical work per cycle (W)
T_{r}	regenerator temperature (K)	W_{c}	compression chamber work per cycle (W)
p	instantaneous value of the pressure (Pa)	W_{e}	expansion chamber work per cycle (W)
p_{m}	mean pressure over the complete cycle (Pa)	X	dead space ratio
P	power (W)	Θ	amplitude of the oscillating movement (rad)
P_{e}	engine power (W)	β	phase angle (rad)
P_{i}	power due to the moving parts inertia (W)	θ_{p1}	angular movement of the piston 1 (rad)
P_{p}	power due to the cycles pressure (W)	θ_{p2}	angular movement of the piston 2 (rad)
Q_{c}	heating capacity (W)	Γ	torque (N m)
V_{c}	compression space instantaneous volume (m^3)	Γ_{e}	engine torque (N m)
V_{k}	compression space heat exchanger volume (m^3)	Γ_{i}	inertia torque (N m)
V_{r}	regenerator space volume (m^3)	Γ_{p}	pressure torque (N m)
V_{h}	expansion space heat exchanger volume (m^3)	η	efficiency
V_{e}	expansion space instantaneous volume (m^3)	η_{Carnot}	Carnot efficiency
V_{swc}	compression space swept volume (m^3)	η_{exp}	experimental efficiency

coupled to the working pistons. Stirling machines use common transmission mechanisms, such as a swash plate [7] or a wobble mechanism [8]. While Clucas [8] obtained an overall efficiency of 85%, the measured values by Haywood attained values no greater than 80% [3]. The elimination of the transmission mechanism makes it possible to avoid the mentioned mechanical losses, resulting in an important increase in the overall efficiency in comparison with conventional Stirling heat pumps.

Although conventional vapour compression heat pumps are known to allow primary energy savings and therefore reduced CO_2 emissions [9], the working fluids used in this technology, chlorofluorocarbons, have been identified as devastating pollutants for the environment [10]. In contraposition, Stirling technology-based heat pumps have been identified as an alternative efficient technology [11] and use environmentally friendly working gases, such as nitrogen or helium [12].

In this study the working mode of an innovative oscillating Stirling heat pump is described. The dynamics of the system are analysed, and a methodology to obtain a preliminary design of this type of heat pump is developed. The methodology is based on a model in which the thermodynamics of the heat pump are coupled with the dynamics via an isothermal model. The stability of operation is also explored by considering the start-up and load condition changes.

2. Thermodynamics of Stirling machines

The oscillating Stirling heat pump is based on the volume variation produced by two oscillating pistons with a phase angle difference between their motions. A double-acting configuration is developed, and, among the different possibilities that can obtain a Stirling cycle, the variant shown in Fig. 1 is analysed. The four cycles composing the heat pump have a $\pi/2$ delay as in the case of the Stirling type developed by Siemens [13].

The full operational cycle can be described via the diagrams of the four distinctive processes shown in Fig. 2. The thermodynamic processes on-going in each cycle are summarized in Table 1.

To evaluate the pressure acting over the pistons, an isothermal model is used as proposed by Berchowitz [5] and Redlich [6]. This model, based on Schmidt analysis, assumes that all of the gas in the

hot space is at temperature T_{h} and all of the gas in the cold space is at T_{k} . The influence of the employed working gas can be explored by modifying the corresponding gas constant in the next expression, which is derived from the ideal gas law:

$$p(t) = m_{\text{t}}R/[V_{\text{c}}(t)/T_{\text{h}} + V_{\text{k}}/T_{\text{h}} + V_{\text{r}}/T_{\text{r}} + V_{\text{h}}/T_{\text{k}} + V_{\text{e}}(t)/T_{\text{k}}] \quad (1)$$

The temperature in the regenerator is usually considered as:

$$T_{\text{r}} = (T_{\text{h}} - T_{\text{k}})/\ln(T_{\text{h}}/T_{\text{k}}) \quad (2)$$

Considering the amplitude of the movement to be Θ , as shown in Fig. 3, the swept volumes are defined as follows,

$$V_{\text{swe}} = V_{\text{swc}} = (R_2^2 - R_1^2)\Theta L/2 \quad (3)$$

The instantaneous value of compression and expansion spaces may be expressed as:

$$V_{\text{e}} = (R_2^2 - R_1^2)\theta_{\text{p1}}L/2 \quad (4)$$

$$V_{\text{c}} = (R_2^2 - R_1^2)\theta_{\text{p2}}L/2 \quad (5)$$

The angular movement, speed and acceleration of the two pistons are shown in Fig. 4. Assuming the mean pressure of the cycle p_{m} to be equal to the charge pressure, the total mass of gas, m_{t} , in the cycle can be obtained using the following expression:

$$m_{\text{t}} = R[1/(V_{\text{c}}/T_{\text{h}} + V_{\text{k}}/T_{\text{h}} + V_{\text{r}}/T_{\text{r}} + V_{\text{h}}/T_{\text{k}} + V_{\text{e}}/T_{\text{k}})]_{\text{m}}/p_{\text{m}} \quad (6)$$

Using Eqs. (1)–(5), the variation of the pressure along the cycle can be calculated. The pressure evolution for the four cycles composing the heat pump and the resulting pressure over the two pistons is shown in Fig. 5.

In a heat pump application, for a given input work, the maximum possible heat capacity is desired. The work input to the system is the sum of the work supplied to the compression and expansion spaces. Over a complete cycle, this is given by:

$$W = W_{\text{c}} + W_{\text{e}} = \oint p dV_{\text{c}} + \oint p dV_{\text{e}} \quad (7)$$

The heat capacity, Q_{c} , can be related to the input work through the Coefficient of Performance, $\eta = Q_{\text{c}}/W$. In this case, the efficiency of an isothermal model is normally expressed in the form of the Carnot limit between two thermal reservoirs:

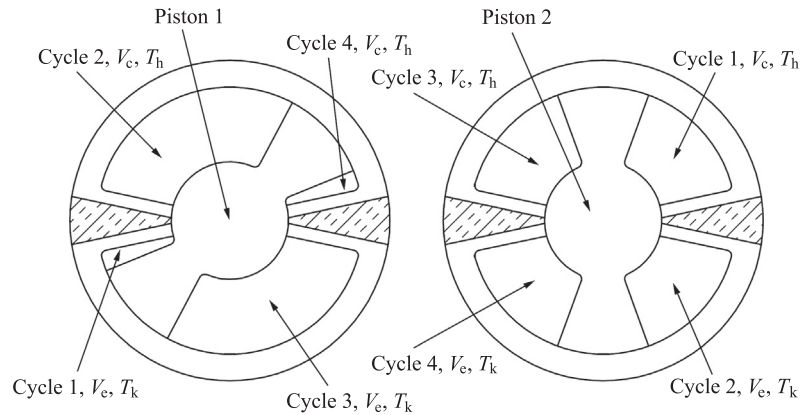


Fig. 1. Overall scheme of the proposed heat pump.

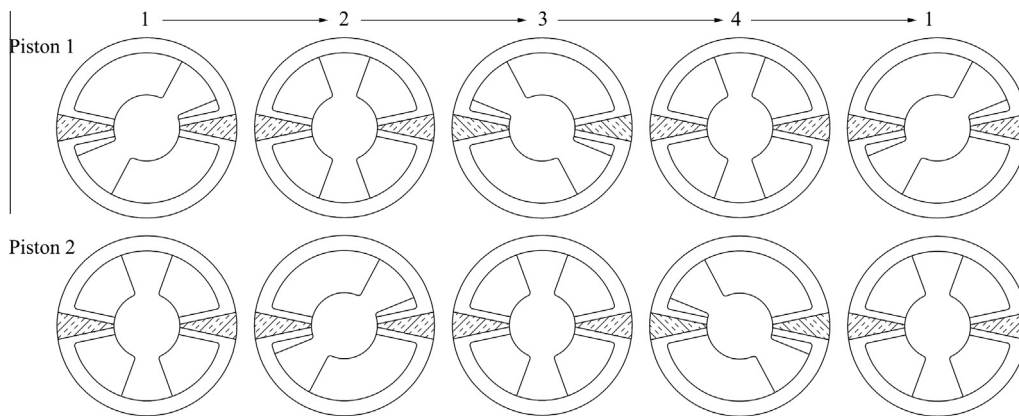


Fig. 2. Movement of the pistons along a cycle.

Table 1

Thermodynamic processes in each cycle along a complete period.

	Cycle 1	Cycle 2	Cycle 3	Cycle 4
Process 1 → 2	Isochoric cooling	Isothermal compression	Isochoric heating	Isothermal expansion
Process 2 → 3	Isothermal expansion	Isochoric cooling	Isothermal compression	Isochoric heating
Process 3 → 4	Isochoric heating	Isothermal expansion	Isochoric cooling	Isothermal compression
Process 4 → 1	Isothermal compression	Isochoric heating	Isothermal expansion	Isochoric cooling

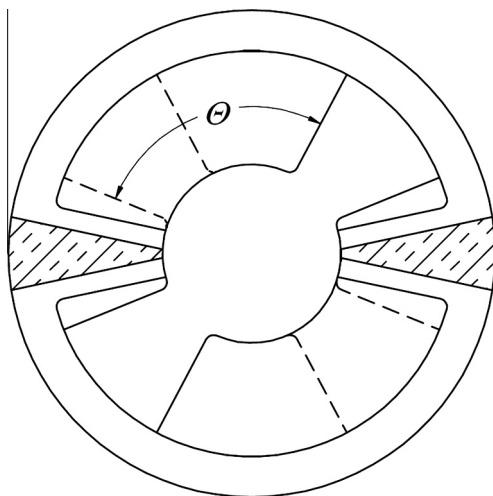


Fig. 3. Total amplitude of the movement.

$$\eta = T_h / (T_h - T_k) \quad (8)$$

2.1. Identification of the values of thermodynamic variables

The complete design of a Stirling based system should be carried out in sequential stages, with each consecutive stage embodying a more rigorous analytical treatment [7]. As the aim of the present work is to provide a methodology to preliminarily size the proposed type of heat pumps, then the design evaluations needed may be simple calculations to ascertain the possible performance of the system. An isothermal analysis together with values available from the literature is proposed for use as design guidelines. The variables defined are: dead volumes, operating temperatures T_h and T_k , mean pressure p_m , swept volumes V_{swe} and V_{swc} , performance η and the required work input W .

To evaluate the dead volumes, usually the dead space ratio is used. This ratio compares the dead volumes with the expansion swept volume:

$$X = V_D / V_{swe} \quad (9)$$

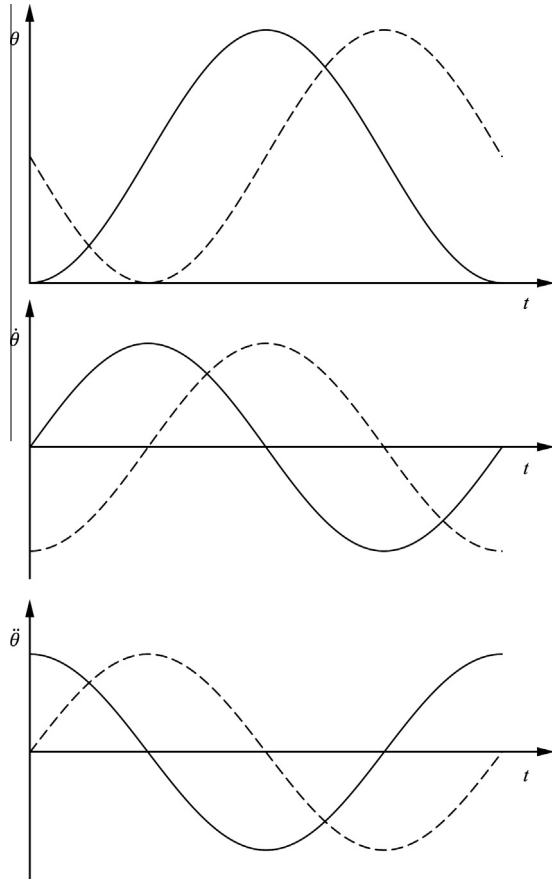


Fig. 4. Angular movement, speed and acceleration.

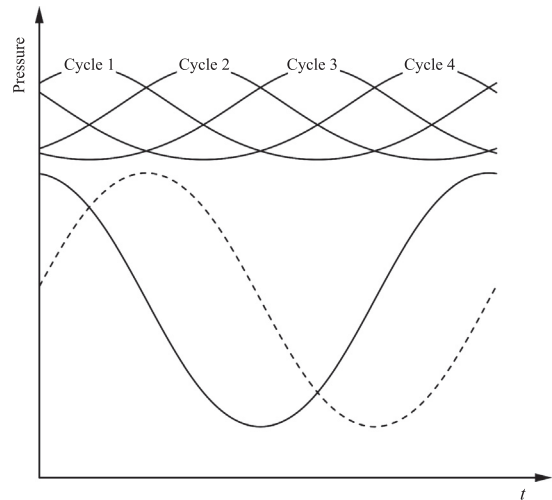


Fig. 5. Pressure evolution along the cycle.

According to Reader [7], the dead volume ratio is usually between 1.3 and 1.7. The values found in the literature show a higher proportion of dead volume. The dead volume of the heat pump prototype developed by Haywood [3], for example, has a value of 2.3, and the Stirling engine developed by Ford-Philips [14] and known as 4-215 has a value of 2.1. Considering the values observed in the literature review, the dead volumes shown in Table 2 are proposed as design guidelines.

Once dead volumes are chosen, the specifications of the application to be developed define the required heating capacity and the operating temperatures.

Table 2
Proposed dead volumes.

Chamber	Value
Expansion chamber	0.15
Hot side heat exchanger	0.3
Regenerator	0.7
Cold side heat exchanger	0.3
Distribution channels	0.3
Compression chamber	0.15
Total dead volume	1.9

The pressure and swept volumes relation is obtained using the isothermal analysis to have a preliminary idea of their value. The swept volumes and mean pressure are inversely proportional, as shown in Fig. 6. In this figure, the heating capacity is kept at a value of 2.1 kW to compare theoretical values with the experimental results obtained by Haywood [3]. This comparison is shown in Table 3.

A starting point to define the mean pressure and swept volumes may be to specify a maximum reference for the pressure, considering that the structural verification depends on this value. The resulting swept volume can be evaluated in relation to the expected size of the machine.

As can be seen in Table 3, the theoretical performance value obtained with the isothermal model is far higher than that obtained experimentally. Similar results have often been discussed in the literature. In the case of a prime mover Stirling application, Reader [7] estimates the actual power output to be 30–50% of the one estimated with the isothermal model.

The little information found in the literature regarding Stirling heat pump applications is summarized in Table 4. These values indicate that, for a heat pump application, the performance is bounded between 2 and 4. Once a performance value is chosen considering this range, the work input to be given by the oscillating machine is obtained as well.

3. Dynamics analysis

Once a preliminary thermodynamic design is selected, it is time to explore the dynamics of the system. The torque due to the cycle pressure and the engine torque will act on the pistons; losses will not be considered. The analysis is focused on piston 1, as the only difference with piston 2 is a delay of 90° in forces and movement.

Applying Newton's second law for rotation over one piston, we have:

$$\Gamma_p(t) + \Gamma_e(t) = I\ddot{\theta}(t) \quad (10)$$

The power to be supplied by the engine can be expressed as:

$$P_e(t) = \Gamma_e(t)\dot{\theta}(t) = (I\ddot{\theta}(t))\dot{\theta}(t) - \Gamma_p(t)\dot{\theta}(t) \quad (11)$$

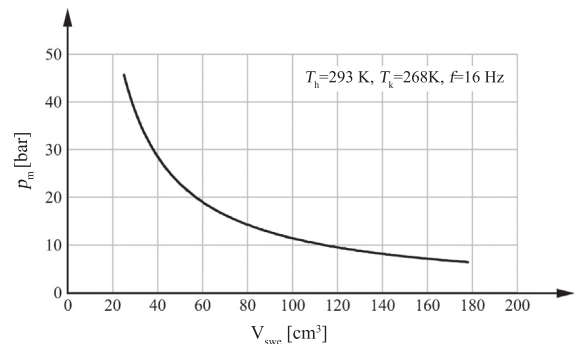


Fig. 6. Expansion swept volume vs. charge pressure for a given heating capacity.

Table 3

Theoretical and experimental efficiency results comparison.

	DH1 prototype				Isothermal model			
	η	Q_c (kW)	V_{swe} (cm ³)	p_m (bar)	η	Q_c (kW)	V_{swe} (cm ³)	p_m (bar)
$T_h = 293$ K, $T_k = 268$ K	2.44	2.11	57.2	25	11.72	2.11	57.2	19.9

Table 4

Achievable efficiencies found in the literature.

	η_{exp}	Q_c (kW)	T_h (K)	T_k (K)	η_{Carnot}	η_{exp}/η_{Carnot} (%)
David Haywood [3]	2.44	2.11	293	268	11.72	20.8
David Haywood [3]	2.77	2.19	293	278	19.53	14.18
STM, Inc. [2]	4.3	15	296	280	18.5	23.2

For the working temperature range of a heat pump, the torques acting over the piston due to the inertia and pressure compensate for each other, as shown in Fig. 7.

Normally, when designing a Stirling machine with a drive mechanism, it is desired to have a perfect balance with the sum of all of the inertia forces in the system equal to zero [15]; in this case, a specific value of inertia is required to compensate pressure forces. This effect is explained as follows.

Considering the torques acting on the piston from position 1 to 2 in Fig. 7, the resulting pressure over the piston will act in the anti-clockwise direction, helping in this way to overcome inertia forces and accelerating the piston in the anti-clockwise direction.

From position 2 to 3 in Fig. 7, the pressure will act in the clockwise direction, slowing down the motion of the piston. At the beginning of this process, the speed of the piston is at its maximum value, and, along the process, this kinematic energy is transferred to the working gas.

Then, from position 3 to 4, the event from position 1 to 2 is repeated: the resulting pressure again accelerates the piston.

3.1. Identification of dynamics analysis variables

The inputs to this analysis will be the piston motion and the resultant pressure acting over the piston. The next analysis may be used to minimize the peak power to be supplied by the oscillating machine according to an inertia value that compensates the pressure torque. The geometry of the piston will be defined as well. Firstly, the variables that influence the torque and power due to pressure and inertia forces are analysed.

The torque over the piston caused by the cycle pressure is defined by the variables defined in the isothermal analysis and by the geometrical parameters of the piston: the area of the piston palette and the distance to the rotation axis.

$$\Gamma_p(t) = p(t)Ad \quad (12)$$

Developing the pressure power expression, it can be seen that it is directly proportional to the frequency of oscillation:

$$\begin{aligned} P_p(t) &= \Gamma_p(t)\dot{\theta}(t) = p(t)Ad \frac{\Theta}{2} \omega \sin(\omega t) = p(t)Ad \frac{\Theta}{2} \omega \sin(\omega t) \\ &= p(t)V_{swe}\pi f \sin(\omega t) \end{aligned} \quad (13)$$

The pressure power is related to the geometry by the definition of the swept volumes, as shown in expression (3).

The torque due to the inertia forces is directly proportional to the inertia I . It also depends on the square value of the frequency of oscillation as $\omega = 2\pi f$.

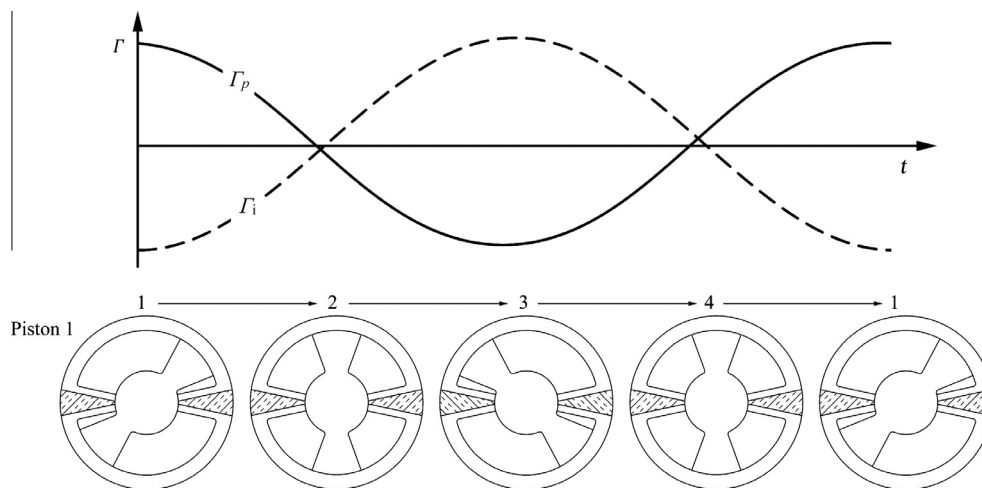
$$\Gamma_i(t) = I\ddot{\theta}(t) = I \frac{\Theta}{2} \omega^2 \cos(\omega t) \quad (14)$$

In the case of the inertia power, it is proportional to I and to the third power of the frequency, as shown in the next expression:

$$P_i(t) = (I\ddot{\theta}(t))\dot{\theta}(t) = I \frac{\Theta^2}{4} \omega^3 \cos(\omega t) \sin(\omega t) \quad (15)$$

The variables to be defined now are the inertia and the working frequency of the heat pump. The working frequency will depend on the oscillating machine, and, once it is selected, the inertia will be defined to minimize the engine peak power. In Fig. 8, the peak value of pressure, inertia and engine powers are shown for a range of frequencies from 0 to 20 Hz. In this figure, the inertia value has been chosen to minimize the peak value of the engine power for a frequency of 16 Hz. A movement amplitude of 150° and the values shown in Table 3 ($T_h = 293$ K and $T_k = 268$ K) have been considered.

Bearing in mind the selected inertia and the expression that defined the swept volumes (3), a preliminary definition of the geometry can be obtained. The peak power to be supplied by the

**Fig. 7.** Pressure and inertia torques along a cycle.

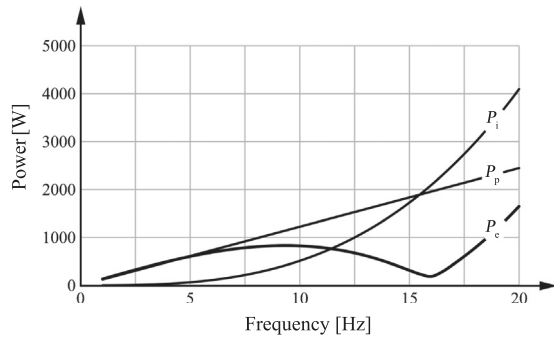


Fig. 8. Pressure, inertia and engine peak powers.

engine is now defined, and, as the chosen inertia considers all of the moving parts, once the pistons' geometry and material are chosen, the rotor inertia will be defined as well. In this way, the required oscillation machine can be evaluated.

4. Proposed design methodology

As a summary, the proposed design methodology will be described briefly. This methodology is based on the isothermal model and the dynamic analysis. A scheme is shown in Fig. 9.

Firstly, the isothermal model variables are defined by considering the requirements of the heat pump to be developed and the information available in the literature. The variables defined with help of the literature are the dead volumes and the performance. Variables such as the working temperatures and heat capacity are defined by the requirements.

Swept volumes and the mean pressure are defined bearing in mind that the structural requirements of the different components depend on the mean pressure and that the size of the system depends on the swept volumes. The relation between these two variables is defined in Fig. 6 for a fixed heat capacity.

Secondly, the dynamic analysis is carried out. The input from the isothermal model is the pressure torque, power and piston motion. In this analysis, the total inertia is defined considering that the pressure torque is compensated by the inertia torques to minimize the peak power to be supplied by the engine. Considering the

obtained inertia and the expression (3) that defines the swept volumes, the overall geometry may be defined.

Lastly, the oscillating machine has to be checked. The input information from the dynamic analysis is: the mean and peak power, working frequency and inertia of the rotor. Based on this information, the oscillating machine is evaluated. If some of the specifications are not possible to achieve, the process is repeated until a feasible solution is found.

5. Stability of operation

Ideally, the proposed design should be valid for the entire frequency range over the start-up and also for a range of working temperatures. The stability of operation will be explored by considering these changes in relation to the load conditions used for the preliminary design of the heat pump. For this aim, a model is developed in MATLAB based on the equations shown previously in Sections 2 and 3.

5.1. Start-up

Ideally, the inertia torque should compensate for the pressure torque over the entire frequency range of the start-up. As the inertia and pressure power are influenced by the frequency at different levels, this ideal condition is not achieved and the required engine power goes over the maximum, as can be seen in Fig. 8.

Although this problem is quite common in Stirling applications, not much information is available in the literature about it. One of

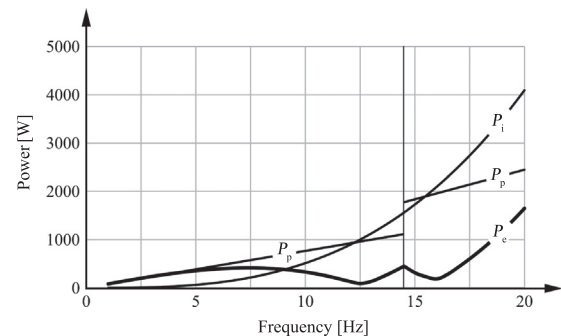


Fig. 10. Pressure, inertia and engine peak powers, start-up proposal.

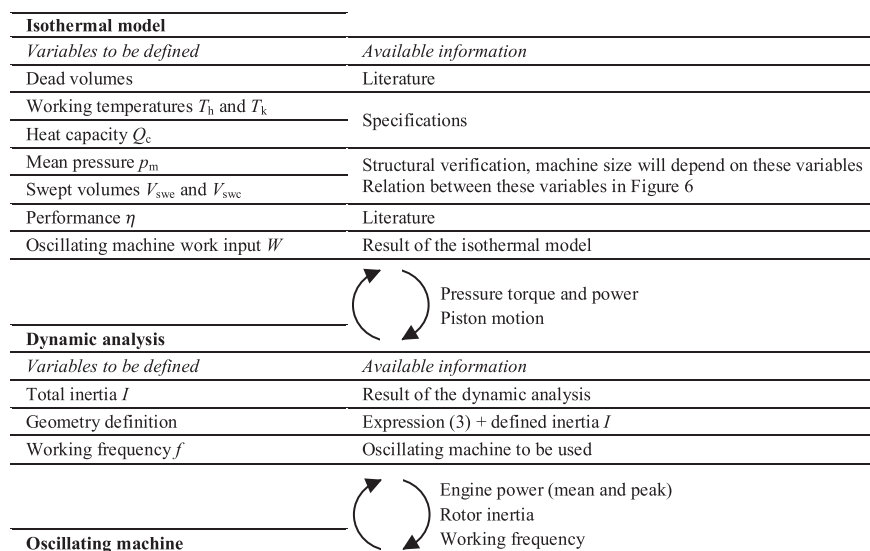


Fig. 9. Proposed design methodology.

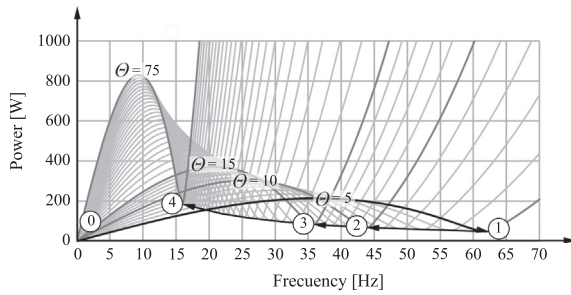


Fig. 11. Engine peak power for different movement amplitudes, start-up proposal.

Table 5
Comparison of mean and peak engine powers for different movement amplitudes.

	f (Hz)	Θ (°)	W_{mean} (W)	W_{peak} (W)
0	0	0	0	0
1	62	5	23.2	48.4
0 → 1	36	5	13.5	214.5
2	44	10	32.8	69
1 → 2				<100
3	36	15	40.4	85.3
2 → 3				<100
4	16	75	90	187
3 → 4				<100

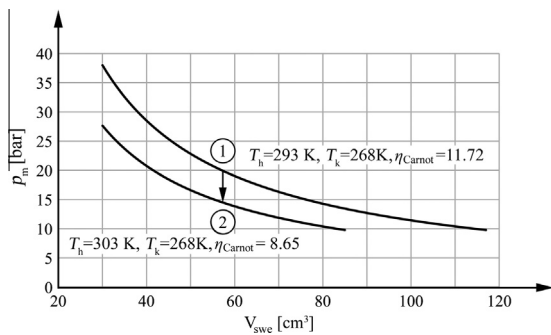


Fig. 12. Expansion swept volume vs. charge pressure for a given input power.

the possible solutions to facilitate motor operation during the starting period is to complement the working spaces with a starting volume, enlarging in this way the volume to be compressed. The volume compression ratio will be reduced by increases in dead space, and this will affect the cyclic pressure swing. The result over the peak power to be supplied by the engine is shown in Fig. 10 in comparison with Fig. 8. This system was used in the Stirling refrigerator “A-machine” developed by Philips [12].

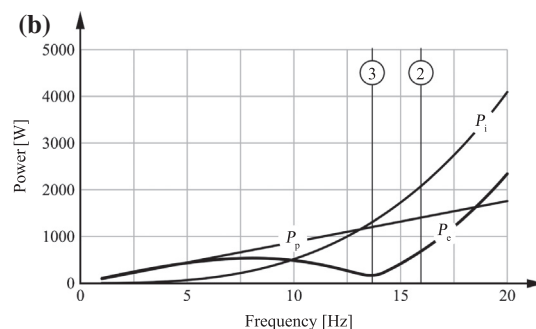
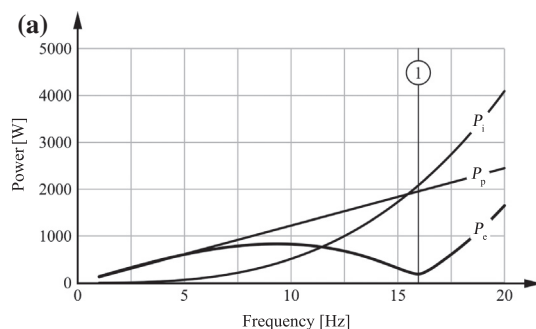


Fig. 13. Pressure, inertia and engine peak powers, working temperatures varied 1.

Table 6
Comparison of mean and peak engine powers for different working modes.

	T_h (K)	T_k (K)	p_m (bar)	f (Hz)	V_{swe} (cm³)	W_{mean} (W)	W_{peak} (W)
1	293	268	19.9	16	57.2	90	187
2	303	268	14.5	16	57.2	90	690
3	303	268	14.5	13.5	57.2	75	165
4	303	268	16	14.5	57.2	90	190
5	303	268	14.5	16	57.2	90	188

The configuration on which the present study is based offers a start-up possibility with no need of complementary volumes or additional devices. This start-up is based on a variation of the movement amplitude and working frequency. In Fig. 11, the maximum power supplied by the oscillating machine is plotted against the working frequency for different movement amplitudes.

It is shown in Fig. 11 that, in the case of small movement amplitudes, the maximum power to be supplied by the oscillating machine goes over a maximum value, which is smaller than that of the other cases with higher values of movement amplitude. Once this tendency is identified, the next start-up logic is proposed: initially, a small movement amplitude will be used, and the frequency will be increased until the minimum identified in Fig. 11 with number 1 is reached. From this point on, the movement amplitude will be increased together with a reduction of the working frequency. Table 5 provides the values of the maximum power supplied by the oscillating machine for the points identified in Fig. 11.

5.2. Variation of working temperatures

A variation of the working temperatures affects directly the performance of conventional and Stirling heat pumps, as can be directly observed in Eq. (8). This circumstance may be encountered, for example, if the required output temperature of the heat pump is modified. The behaviour of the proposed system is also affected as described in the following. Fig. 12 shows how the mean pressure and efficiency vary if the working temperatures are modified. In this case, the working input value is kept constant at a value of 90 W for each engine (total input work of 180 W) and the work frequency at a value of 16 Hz.

There would be two possibilities to compensate the inertia and pressure torques due to a new condition of working temperatures. The first one would be to modify the mean pressure and the frequency.

Point one in Fig. 13(a) shows the design point; as can be seen, the engine peak power is in the optimal value. Point 2 in Fig. 13(b) shows how this optimal situation is modified when the working temperatures are changed: the mean pressure has been decreased to maintain constant input power. As the inertia and

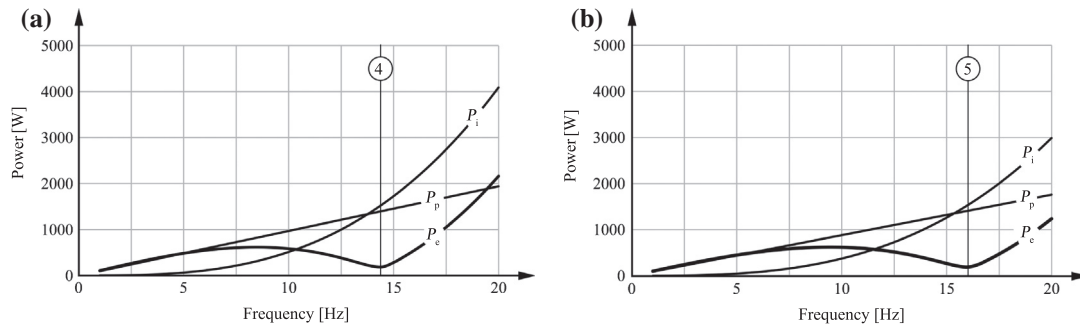


Fig. 14. Pressure, inertia and engine peak powers, working temperatures varied 2.

pressure torques are not in phase now, the engine peak power increases, as can be seen in Table 6.

Now, to reduce the engine peak power, the frequency could be reduced, but the mean work supplied by the engine would be quite reduced, as shown in Table 6, situation 3. The best option would be to find a new relation between mean pressure and frequency to minimize the engine peak power and keep the mean power close to the desired one; this is shown in situation 4, Table 6 and also in Fig. 14(a).

The second option to compensate the inertia and pressure torques would be to modify the inertia of the system. This may be done if a flywheel is considered in the design. The situation achieved with this option is shown in Fig. 14(b) and in Table 6.

6. Conclusions

In summary, the working mode of an innovative oscillating Stirling heat pump is described, and a methodology has been developed to obtain the preliminary design of the basic variables of this type of Stirling heat pump. To develop this guideline, the information available in the literature and a study of the dynamics of the system have been used. The following has been shown:

1. Existence of an optimal working condition that relates mechanical dynamics and thermodynamics. This working condition is achieved with a certain value of the inertia, obtaining a minimum of the peak power to be supplied by the oscillating machine.
2. Dead volumes and performance values are identified using the available information in the literature.
3. A quick preliminary design methodology has been proposed based on the isothermal model and dynamic analysis.
4. System stability is analysed during the start-up. The observed power peak can be reduced by complementing the working spaces with a starting volume. In this way, the volume compression ratio will be reduced by increases in dead space, which will affect the cyclic pressure swing.
5. A start-up logic has been proposed that avoids the use of the complementary volumes or additional devices usually used in Stirling applications. This start-up is based on a variation of the movement amplitude and working frequency.
6. Changes in working temperature are analysed, and the consequences over the system are explained. Two possibilities to compensate the imbalance due to this situation are described:

pressure and working frequency adjustment and modification of the total inertia.

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